

Charging Ahead

- Optimal Location of Wireless Power Transfer Systems to Electrify Roads in Urban Environments -

Thomas Byrne ¹⁾ Yudai Honma ²⁾

1) University of Strathclyde, Glasgow, United Kingdom

E-mail: tom.byrne@strath.ac.uk

2) University of Tokyo, Tokyo, Japan

E-mail: yudai@iis.u-tokyo.ac.jp

ABSTRACT: The popularization of electric vehicles (EVs) is limited by their driving range and long charging times. To address this, in-motion charging solutions are currently attracting attention as a new power supply system. In-motion charging infrastructure such as wireless power transfer systems (WPTSs) have coils embedded under the road to transfer power from the WPTSs to EVs while driving. However, the main drawback of this technology is their large investment, especially in supporting the long-distance trips of EVs on expressways. Therefore, this study proposes new models for determining the optimal location of in-motion charging infrastructure for maximized total feasible flow demand or minimised external power requirements for the entire system. We observe that in-motion charging has strong potential as an EV power supply system in terms of coverage and economic rationality. In particular, in-motion charging has economic rationality not only in busy networks but also in sparsely populated networks that connect urban and rural areas. Thus, this study clarifies the important insights of in-motion charging infrastructure planning in improving their effectivity to narrow down the demand and ensure the flexibility in the locations of implementations of this technology.

KEYWORDS: dynamic routing, in-motion charging, wireless power transfer systems, combinatorial optimisation, continuous demand, computational geometry

1. INTRODUCTION

Most experts agree that the electrification of the transportation sector will be vital in our efforts to stem climate change. Indeed, if all cars on the road became electric, we could cut almost one-fifth of global emissions. To this end, the UK government (and many like it) has announced a ban on the sale of new petrol and diesel cars after 2035%. However, currently, fewer than 1% of cars on UK roads are powered entirely by electricity, with similar stories in most other countries.

The two widely accepted chief barriers to switching to an electric vehicle (EV) for private, commercial, and public transport are cost and “range anxiety”. Fortunately, a recently-developed technology solves both: a wireless power transfer system (WPTS) on which vehicles can charge while in motion. By directly and efficiently receiving power while moving along an “electric road”, battery size as well as dedicated charging time and space can be saved. This revolutionary technology is being widely heralded as the future of transport. However, it is of little value if not

effectively implemented. The question of what to electrify remains (WPTSs are prohibitively expensive) and it is this challenge to which this presentation rises.

Given the novelty of this inductive power transfer technology, not surprisingly there is little literature on the optimal deployment of a WPTS. Most such models have the constraint that the vehicle must travel a predetermined route. Most notably, Ko et al. (2015) developed a mixed-integer nonlinear model for an electric shuttle bus with both WPTS lanes and their length as decision variables. Chen et al. (2016) developed a user equilibrium model for EV drivers’ choice of routes and Riemann et al. (2015) produced a flow-capturing placement problem, also assuming that drivers choose routes by considering congestion and WPTS placement.

To date, the most advanced models are those by Honma et al. (2024), crucially addressing non-guided WPTSs for millions of users on expressways. Here, a new mixed-integer programming model was proposed to determine the optimal location of WPTSs in order to maximise total feasible demand flow on a transport

network. This flow-capturing model for WPTS locations focused on long-distance trips on expressways, considering the installation of WPTSs as continuous variables (and observed that a WPTS has a strong potential as an electric vehicle power supply system in terms of coverage and economic rationality).

An alternative focus which has high demand for such a charging infrastructure is in urban environments. Given the slower speeds travelled upon city streets (often with stationary traffic) and reduced area compared to expressways, this represents an application with lower investment cost and likely higher utilisation. An early breakthrough into this area by Honma et al. (n.d.) produced another mixed-integer programming model incorporating comprehensive traffic data to determine the optimal locations and lengths of WPTS installations. It was demonstrated that, by strategically placing WPTS infrastructure, an entire city's charging needs could be satisfied by electrifying less than 2% of the region's total road length. This research underscores the viability of WPTS in promoting sustainable urban mobility, providing key insights into the practical deployment and economic considerations of EV infrastructure.

It is from here that we take our inspiration; we will design and implement optimisation models to identify optimal segments of a general urban transport network for electrification, taking into account the continuous distribution of population and the behaviours of the transport infrastructure's users. The goal, as before, is to maximise the number of EV users of the road network. In order to design such a robust network model, we exploit the grid-like structures found naturally in most urban environments and utilise geometric results to quantify the benefit of electrifying select edges, all while incorporating re-routing behaviours seldom approached in previous studies -- we adjust our routes in order to pass a petrol station, so why should the same not be true for passing over an electric road?

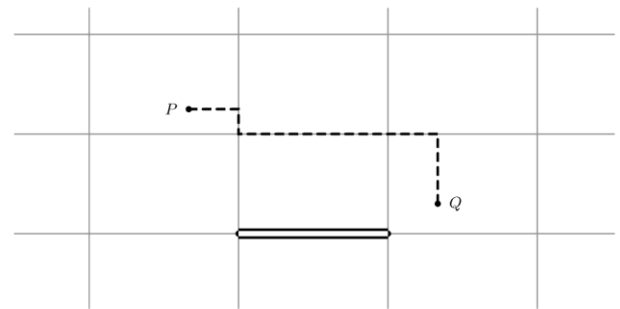
2. MODEL FORMULATION

We assume that an urban population is distributed continuously and uniformly across a region of interest and upon this region there exists a road grid network of blocks of horizontal side length L and vertical side length H . Travel is only permitted through a block if the origin or destination of a route is located within a block (for all other travel the main road grid network is used) and paths across a block are dictated by the l_1 metric.

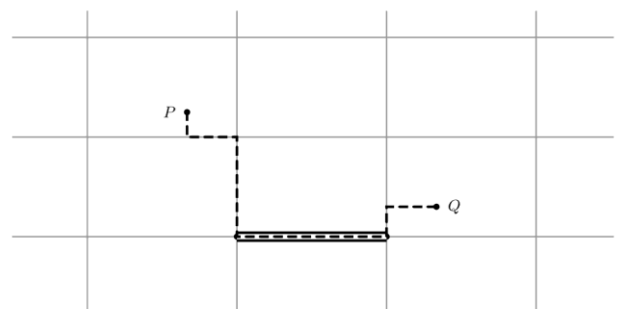
We assume that the speed of travel across a block is V_R while the speed of travel upon the road network is V_G . (It is most likely that the road network represents a high(er)-speed transport

infrastructure so $V_G > V_R$, though this distinction is interesting to explore nonetheless.) Owing to this constant speed assumption (i.e. power usage per unit of distance travelled is constant), the energy cost of travel on the grid is directly proportional to the time spent travelling so these two become interchangeable.

We consider the locating of road improvements on edges of the road network, with one road improvement (WPTS) permitted per grid edge. We assume that each route receives a fixed energy cost/time deduction of T per unique road improvement edge visited in its entirety (note that journeys cannot obtain infinite time improvements from going back and forth along one road improvement). Figure 1 provides a simple illustration of the continuously distributed population, grid network, and potential journeys under consideration. Supposing that journeys occur between all points within the space equally, we ask along which edges of the grid we should place a road improvement so that the interconnectivity of the city is optimised. That is, which locations of N road improvements will reduce the overall energy consumption of the city.



(a) Shortest path not using road improvement.



(b) Shortest path using road improvement.

Fig. 1 Two paths to consider for the minimum power path between points P and Q .

Equivalently, we want to minimise the average power used for journeys between points in the city. For a city \mathcal{P} with a “distance” measure $d(*,*)$ and assumed uniform population density, the

average shortest distance between points in the city can be expressed as $ASD(\mathcal{P}) = \frac{1}{Area(\mathcal{P})} M(\mathcal{P})$ where

$$M(\mathcal{P}) = \int_{P \in \mathcal{P}} \int_{Q \in \mathcal{P}} d(P, Q) dQ dP$$

After introducing WPTSs, the average shortest distance between points in \mathcal{P} ($ASD_{WPTS}(\mathcal{P})$) is proportional to

$$M_{WPTS}(\mathcal{P}) = \int_{P \in \mathcal{P}} \int_{Q \in \mathcal{P}} d_{WPTS}(P, Q) dQ dP$$

where $d_{WPTS}(P, Q)$ is the power required to travel across \mathcal{P} from P to Q in the presence of the WPTSs. We can formulate this as

$$\begin{aligned} M_{WPTS}(\mathcal{P}) &= \int_{P \in \mathcal{P}} \int_{Q \in \mathcal{P}} d_{WPTS}(P, Q) - d(P, Q) dQ dP \\ &\quad + \int_{P \in \mathcal{P}} \int_{Q \in \mathcal{P}} d(P, Q) dQ dP \\ &= \int_{P \in \mathcal{P}} \int_{Q \in \mathcal{P}} d_{WPTS}(P, Q) - d(P, Q) dQ dP \\ &\quad + M(\mathcal{P}) \end{aligned}$$

Therefore, to evaluate the improvement in connectivity after the addition of WPTSs we need only look at the reduction in time for pairs of points between which the shortest path now uses a WPTS.

3. RESULTS

Firstly, how any P and Q not in the same column or row as one another, the minimum power path between P and Q (not considering a WPTS) has power $l_1(P, Q)$. Therefore, for the majority of journeys, the road network does not affect the power of minimum power journeys – journeys behave as they would in the rectilinear plane.

Our preliminary focus is when a minimum power path uses \mathcal{L} . Since any path through multiple WPTSs can be viewed as a concatenation of separate paths through single WPTSs, our primary results consider one WPTS \mathcal{L} which, for ease of expression, runs horizontally in the grid.

Proposition: For any P, Q not in the same column as \mathcal{L} , the minimum power path from P to Q which uses \mathcal{L} enters \mathcal{L} at A and leaves \mathcal{L} at B where $A = \operatorname{argmin}_{X \in \mathcal{L}} l_1(P, X)$ and $B = \operatorname{argmax}_{X \in \mathcal{L}} l_1(P, X)$.

Therefore, for any origin point P , we need only compare the power of its paths not using a WPTS to the power of its paths using \mathcal{L} leaving from the point on \mathcal{L} furthest from the origin point. That is, the shortest path to destination Q using the WPTS follows the

l_1 -shortest path from P to the furthest end of \mathcal{L} and then the l_1 -shortest path from this end of \mathcal{L} to Q .

This prompts the following very valuable definition: define the expedited space of C to be

$$B_T^{add, \leq}(C, D) = \{X \in \mathbb{R}^2 \mid l_1(C, X) + T \leq l_1(D, X)\}.$$

The expedited space of C to D is the area, bounded by the additive bisector

$$B_T^{add}(C, D) = \{X \in \mathbb{R}^2 \mid l_1(C, X) + T = l_1(D, X)\}$$

containing C . The expedited space of C to D represents all points from which the shortest path to D would visit C for a reward of $T - l_1(C, D)$. [Note, the more negative the reward, the more attractive the detour is.]

Theorem: For origin P not in the same column as \mathcal{L} , let A and B be the closest and furthest endpoints of \mathcal{L} to P respectively. The minimum power paths from P to Q (not in the same column or row as P) travel via \mathcal{L} if and only if $Q \in B_{l_1(P, B) - T}^{add, \leq}(B, P)$.

Thus, the key to understanding route choice from any point is understanding the expression of the expedited space of the furthest point on the WPTS to that point. Outside of the column containing \mathcal{L} , these have six unique forms according to the form that their perimeter (the additive bisector) takes (Figure 2 shows the six unique forms an additive bisector can take depending on the values of T and $l_1(C, D)$, with the sixth being the empty cell if $|T| > l_1(C, D)$).

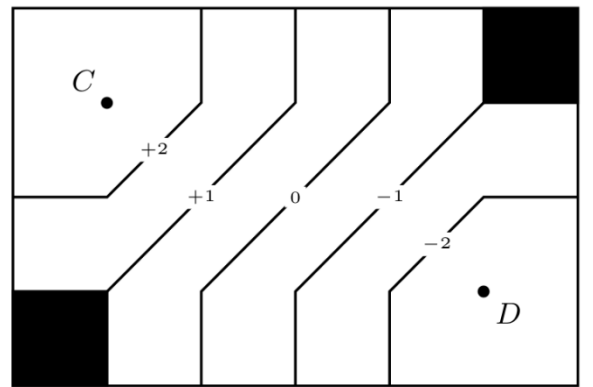
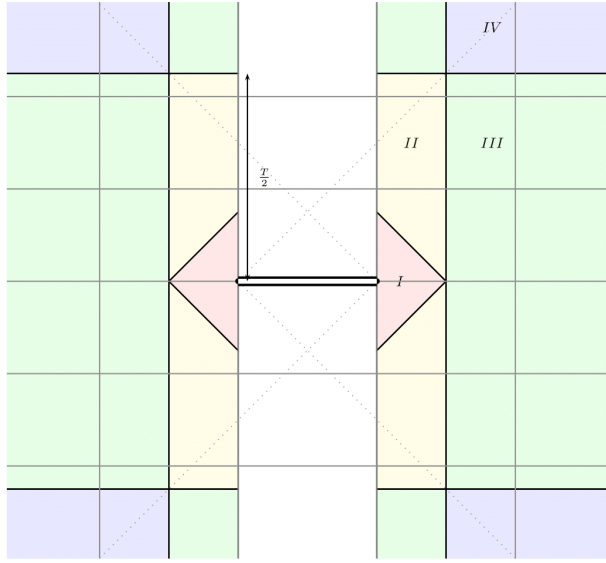
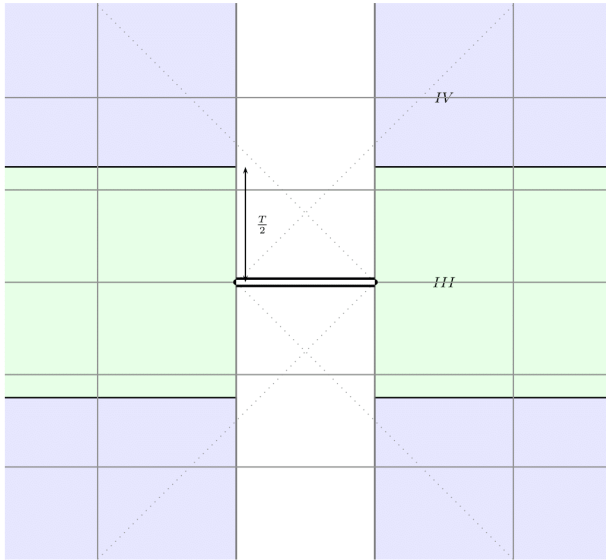


Fig. 2 Six unique forms of $B_T^{add}(C, D)$.

From this we can explore the separate cases producing each structure of the expedited space. The results of this analysis are shown in Figure 3.



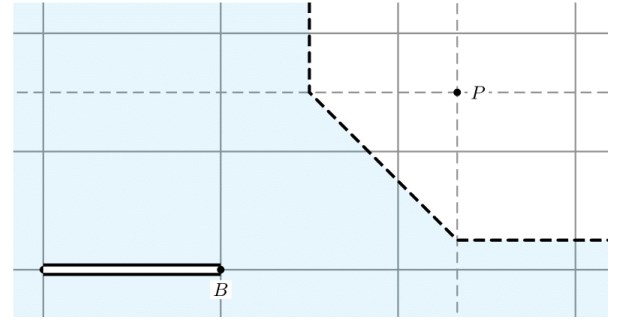
(a) $T > 2L$.



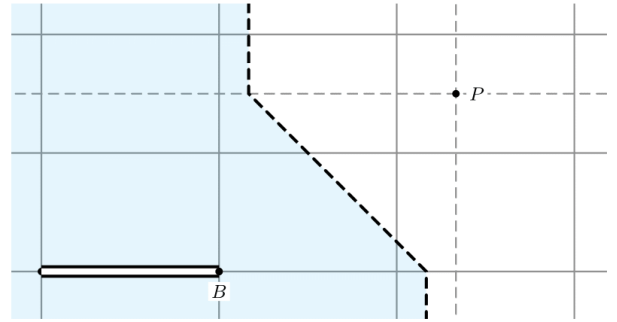
(b) $T \leq 2L$.

Fig. 3 The partition of the space into cells which produce the same expedited space.

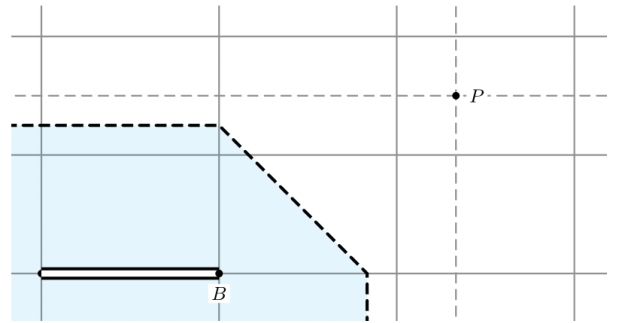
The expedited space for starting points P in each of these partitioned cells can be seen in Figure 4, where the expedited space for P in partition cell I is the entire space (not pictured).



(a) $P \in II$.



(b) $P \in III$.



(c) $P \in IV$.

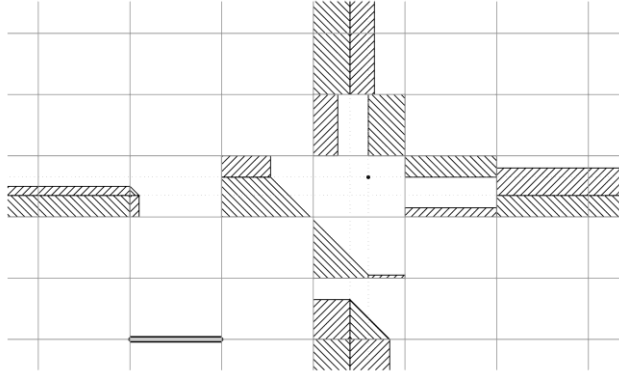
Fig. 4 The expedited space for P in different partitioned cells according to Fig. 3.

Note, similar results concerning whether \mathcal{L} is used exist for when Q is in the same column/row as P and when P is in the same column as \mathcal{L} - however, there is added complication here. For any P and Q in the same n^{th} column (or row) as one another, the minimum power path between P and Q (not considering a WPTS) has power

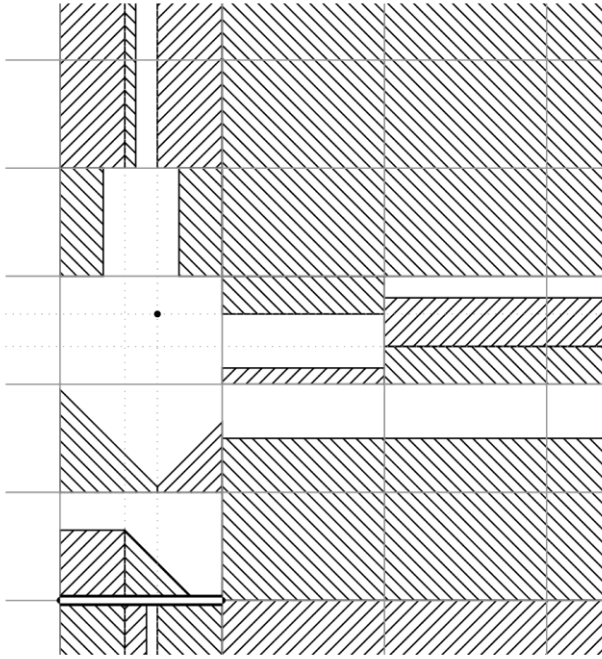
$$\begin{aligned} &|p_y - q_y| + \min\{p_x - (n-1)l + q_x - (n-1)l, nl - p_x + nl - q_x\} \\ &= |p_y - q_y| + \min\{p_x + q_x - 2(n-1)l, 2nl - p_x - q_x\}. \end{aligned}$$

Therefore, for fixed P , considering Q in the same column as P , we must consider separately Q with $q_x \leq (2n-1)l - p_x$ and Q with $q_x > (2n-1)l - p_x$.

Details of the separate cases will be presented during the conference presentation but the partition of the space into regions which give the same structure of the expedited space can be found displayed in Figure 5.



(a) Within the same row or column as P .



(b) P within the same column as L .

Fig. 5 The partition of the space into cells which produce the same expedited space for origin and destination points within the same row or column and for origin points in the same column as L .

Now knowing, from any P , which points Q are reached using the WPTS, we may compute the overall connectivity

$\int_{P \in \mathcal{P}} \int_{Q \in \mathcal{P}} d_{WPTS}(P, Q) - d(P, Q) dQ dP$. Given that the vertices of the expedited space are linear in the coordinates of the endpoints of \mathcal{L} , we obtain a quartic in n and m (for a WPTS on

the lower horizontal edge of the $n^{th} \times m^{th}$ block). Minimising this for n and m provides the optimal position for a WPTS.

These expedited spaces unlock the measuring and assessment of the utility and reach of a proposed in-motion charging installation within an area with an existing developed transport network. This can aid in evaluating not only the effectiveness but the equity of a plan of WPTSs, choosing infrastructure upgrades to minimise external energy requirements or maximise access to in-motion charging. More detailed analysis of our findings, including the extension to multiple WPTSs, will be presented and discussed at the conference.

REFERENCES

- (1) Chen, Z., He, F., & Yin, Y. 2016. Optimal deployment of charging lanes for electric vehicles in transportation networks. *Transportation Research. Part B: Methodological*, 91, 344–365.
- (2) Honma, Y., Hasegawa, D., Hata, K., Zhou, X. S., Kubo, M., & Oguchi, T. Enabling Infinite Drive: Optimal Location of In-Motion Wireless Power Transfer Systems for Trips in Urban-scale Region by Electric Vehicles. *Transportation Research Board Annual Meeting* (under review).
- (3) Honma, Y., Hasegawa, D., Hata, K., & Oguchi, T. 2024. Locational Analysis of In-motion Wireless Power Transfer System for Long-distance Trips by Electric Vehicles: Optimal Locations and Economic Rationality in Japanese Expressway Network. *Networks and Spatial Economics*, 24(1), 261–290.
- (4) Ko, Y. D., Jang, Y. J., & Lee, M. S. 2015. The optimal economic design of the wireless powered intelligent transportation system using genetic algorithm considering nonlinear cost function. *Computers and Industrial Engineering*, 89, 67–79.
- (5) Riemann, R., Wang, D. Z. W., & Busch, F. 2015. Optimal location of wireless charging facilities for electric vehicles: Flow-capturing location model with stochastic user equilibrium. *Transportation Research. Part C: Emerging Technologies*, 58, 1–12A.